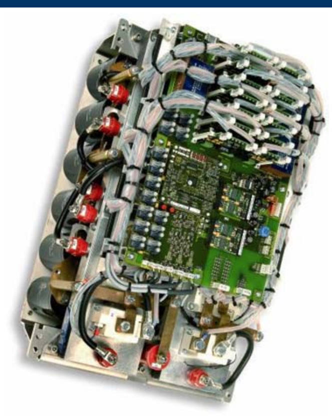
FUNDAMENTALS OF ENGINEERING (FE) EXAMINATION REVIEW



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ELECTRICAL ENGINEERING

Charles A. Gross, Professor Emeritus
Electrical and Comp Engineering
Auburn University Broun 212
334.844.1812
gross@eng.auburn.edu

EE Review Problems

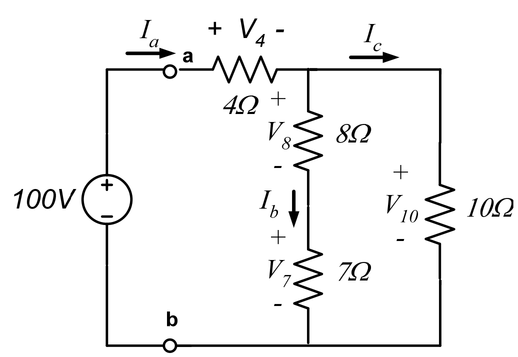
1. dc Circuits
2. Complex Numbers
3. ac Circuits
4. 3-phase Circuits

We will discuss these.

1st Order Transients
Control
Signal Processing
Electronics
Digital Systems

We may discuss these as time permits

1. dc Circuits:



Find all voltages, currents, and powers.

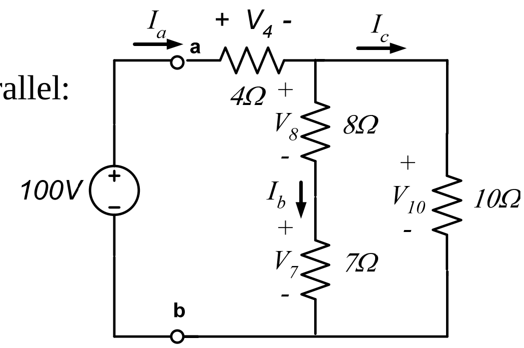
Solution

The 8Ω and 7Ω resistors are in series:

$$R1 = 8 + 7 = 15\Omega$$

R1 and 10Ω are in parallel:

$$R2 = \frac{1}{\frac{1}{10} + \frac{1}{R1}} = \frac{10(R1)}{10 + R1} = 6\Omega$$



Solution

4Ω and R2 are in series:

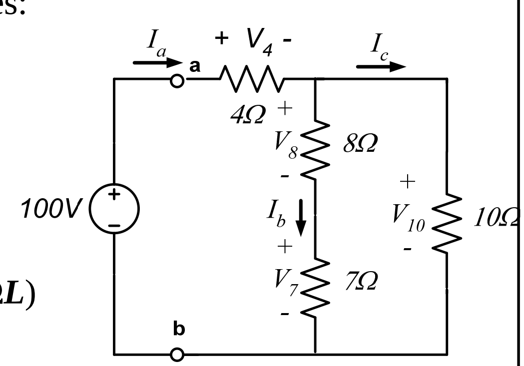
$$R_{ab} = 4 + R2 = 10\Omega$$

ΩL:

$$I_a = \frac{V_{ab}}{R_{ab}} = \frac{100}{10} = 10A$$

$$V_4 = 4 \cdot I_a = 40V \quad (\Omega L)$$

$$V_{10} = 100 - 40 = 60V \quad (KVL)$$

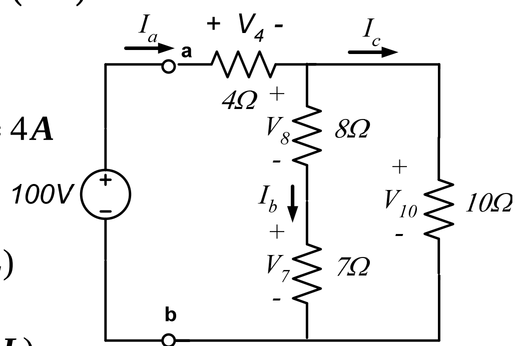


Solution

$$I_c = \frac{V_{10}}{10} = \frac{60}{10} = 6A \quad (\Omega L)$$

$KCL:$

$$I_b = I_a - I_c = 10 - 6 = 4A$$



$$V_8 = 8 \cdot I_b = 32V \quad (\Omega L)$$

$$V_7 = 7 \cdot I_b = 28V \quad (\Omega L)$$

Absorbed Powers...

$$R_4 \cdot I_a^2 = 4(10)^2 = 400W$$

$$R_{10} \cdot I_c^2 = 10(6)^2 = 360W$$

$$R_7 \cdot I_b^2 = 7(4)^2 = 112W$$

$$R_8 \cdot I_b^2 = 8(4)^2 = 128W$$

$$\text{Total Absorbed Power} = 1000W$$

In General:

$P_{ABS} = P_{DEV}$

(Tellegen's Theorem)

$$\text{Power Delivered by Source} = V_s \cdot I_a = 100(10) = 1000W$$

2. Complex Numbers

Consider $x^2 - 2x + 5 = 0$

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(5)}}{2 \cdot 1} = \frac{2 \pm \sqrt{-16}}{2}$$

$$= 1 \pm \frac{4\sqrt{-1}}{2} = 1 \pm 2\sqrt{-1}$$

The numbers "$1 \pm 2\sqrt{-1}$" are called ***complex numbers***

The "I" (j) operator

Math Department....

Define i $= \sqrt{-1}$

$x = 1 \pm 2i$

ECE Department

Define j $= \sqrt{-1}$

$x = 1 \pm j2$

We choose ECE notation! Terminology…

Rectangular Form.....

$\bar{Z} = X + jY =$ a complex number

$X = \Re e\left(\bar{Z}\right) =$ real part of $\bar{Z}$

$Y = \mathrm{Im}\left(\bar{Z}\right) =$ imaginary part of $\bar{Z}$

Polar Form

Math Department.....

$\bar{Z} = R \cdot e^{\theta i} =$ a complex number

$R = |\bar{Z}| =$ modulus of $\bar{Z}$

$\theta = \arg(\bar{Z}) =$ argument of $\bar{Z}$ (radians)

ECE Department.....

$\bar{Z} = Z\angle\theta =$ a complex number

$Z = |\bar{Z}| =$ magnitude of $\bar{Z}$

$\theta = \text{ang}(\bar{Z}) =$ angle of $\bar{Z}$ (degrees)

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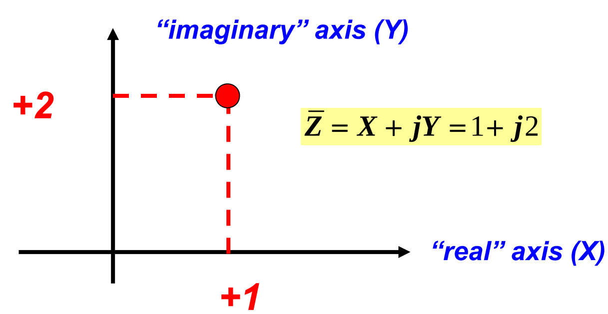
The Argand Diagram

It is useful to plot complex numbers in a 2-D cartesian space, creating the so-called Argand Diagram (Jean Argand (1768-1822)).



$\bar{Z} = X + jY = 1 + j2$

Summer 2008

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Conversions

Retangular $\rightarrow$ Polar

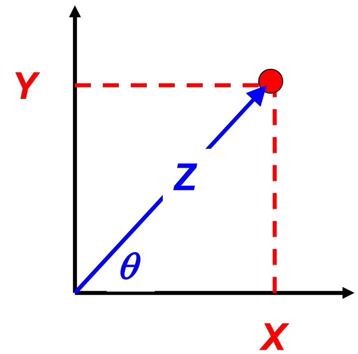
$$Z = \sqrt{X^2 + Y^2}$$

$$\theta = \tan^{-1}\left(\frac{Y}{X}\right)$$

Polar $\rightarrow$ Retangular.....

$$X = Z \cdot \cos(\theta)$$

$$Y = Z \cdot \sin(\theta)$$



Example: $\bar{Z} = 3 + j4$

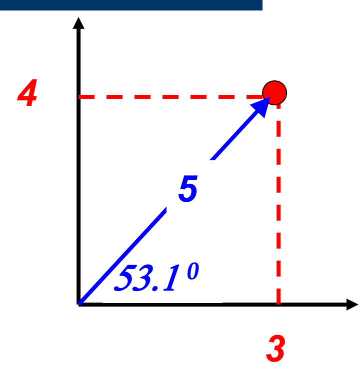
$$X = \Re e\left(\bar{Z}\right) = 3$$

$$Y = \mathrm{Im}\left(\bar{Z}\right) = 4$$

Rect $\rightarrow$ Polar.....

$$Z = \sqrt{3^2 + 4^2} = 5$$

$$\theta = \tan^{-1}\left(\frac{4}{3}\right) = 0.9273 \text{ rad} = 53.1^0$$



Conjugate $\bar{Z} = X + jY = Z\angle\theta$

$$\bar{Z}^* = \text{conjugate of } \bar{Z} = X - jY = Z\angle -\theta$$

Example...

$$(3 + j4)^* = 3 - j4 = 5\angle -53.1^0$$

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Addition (think rectangular)

$$\bar{A} = a + jb = A\angle\alpha \quad = 3 + j4 = 5\angle 53.1^0$$

$$\bar{B} = c + jd = B\angle\beta \quad = 5 - j12 = 13\angle -67.4^0$$

$$\bar{A} + \bar{B} = (a + jb) + (c + jd)$$
$$= (a + c) + j(b + d)$$

$$\bar{A} + \bar{B} = (3 + j4) + (5 - j12)$$
$$= (3 + 5) + j(4 - 12) = 8 - j8$$

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Multiplication (think polar)

$$\bar{A} = a + jb = A\angle\alpha \quad = 3 + j4 = 5\angle 53.1^0$$
$$\bar{B} = c + jd = B\angle\beta \quad = 5 - j12 = 13\angle -67.4^0$$

$$\bar{A}\cdot\bar{B} = (A\angle\alpha)\cdot(B\angle\beta)$$
$$= A\cdot B\angle(\alpha+\beta)$$

$$\bar{A}\cdot\bar{B} = (5\angle 53.1^0)\cdot(13\angle -67.4^0)$$
$$= (5)\cdot(13)\angle(53.1^0 - 67.4^0) = 65\angle -14.3^0$$

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Division (think polar)

$$\bar{A} = a + jb = A\angle\alpha \quad = 3 + j4 = 5\angle 53.1^0$$
$$\bar{B} = c + jd = B\angle\beta \quad = 5 - j12 = 13\angle -67.4^0$$

$$\frac{\bar{A}}{\bar{B}} = \frac{A\angle\alpha}{B\angle\beta} = \left(\frac{A}{B}\right)\angle(\alpha-\beta)$$

$$\frac{\bar{A}}{\bar{B}} = \left(\frac{5\angle 53.1^0}{13\angle -63.4^0}\right) = \left(\frac{5}{13}\right)\angle\left(53.1^0 - \left(-67.4^0\right)\right)$$
$$= 0.3846\angle 120.5^0$$

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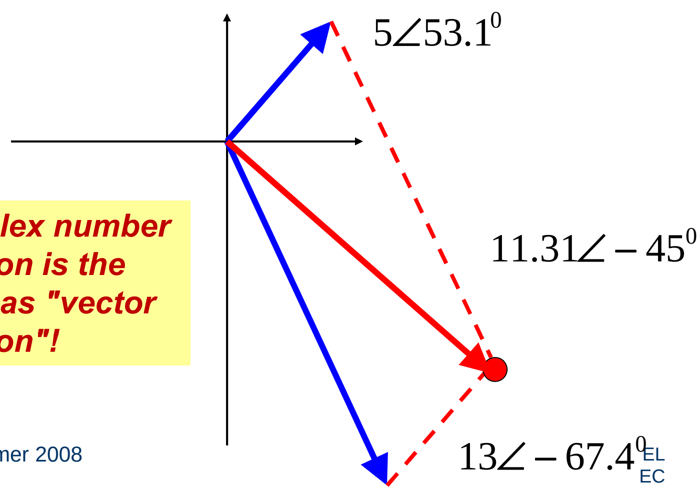
Multiplication (rectangular)

$$\bar{A}\cdot\bar{B} = (a+jb)\cdot(c+jd)$$
$$= (ac-bd)+j(ad+bc)$$

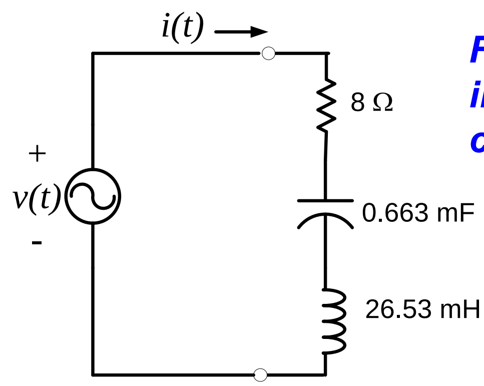
$$\bar{A}\cdot\bar{B} = (3+j4)\cdot(5-j12)$$
$$= (15+48)+j(-36+20)$$
$$= 63-j16 = 65\angle -14.3^0$$

Addition (polar)



Complex number addition is the same as "vector addition"!

3. ac Circuits



Find "everything" in the given circuit.

$$v(t) = 141.4\cos(377t) \quad V$$

Frequency, period

$$v(t) = 141.4\cos(377t) \quad V$$

$$\text{(radian) frequency} = \omega = 377\ rad/s$$

$$\text{(cyclic) frequency} = f = \frac{\omega}{2\pi} = 60\ Hz$$

$$Period = \frac{1}{f} = \frac{1}{60} = 16.67\ ms$$

The ac Circuit

To solve the problem, we convert the circuit into an "ac circuit":

$$R, L, C \text{ elements} \rightarrow \bar{Z}\text{(impedance)}$$

$$v, i \text{ sources} \rightarrow \bar{V}, \bar{I}\text{(phasors)}$$

$$R: \quad \bar{Z}_R = R + j0 = 8 + j0$$

$$L: \quad \bar{Z}_L = 0 + j\omega L = 0 + j(0.377)(26.53) = 0 + j10$$

$$C: \quad \bar{Z}_C = 0 + \frac{1}{j\omega C} = 0 - j\frac{1}{0.377(0.663)} = 0 - j4$$

The Phasor

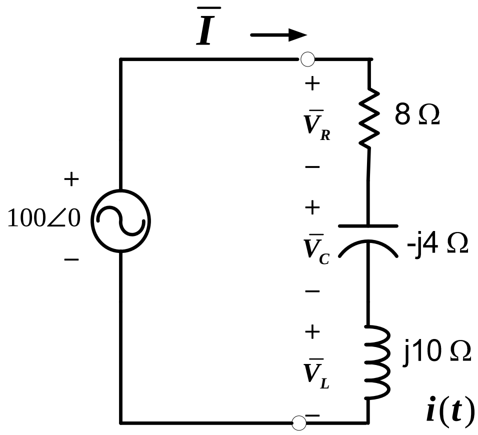
$$v(t) = V_{MAX}\cos(\omega t + \alpha)$$

To convert to a phasor... $\bar{V} = \frac{V_{MAX}}{\sqrt{2}}\angle\alpha$

For example.. $v(t) = 141.4\cos(377t)$

$$\bar{V} = \frac{V_{MAX}}{\sqrt{2}}\angle\alpha = 100\angle 0^o$$

The "ac circuit"



$\Omega L(ac)$:

$$\bar{I} = \frac{\bar{V}}{\bar{\bar{Z}}}$$

$$= \frac{100}{8 + j10 - j4}$$

$$= 10\angle -36.9^o$$

$$i(t) = 14.14 \cdot \cos(377t - 36.9^o)$$

Solving for voltages

$$\bar{V}_R = \bar{Z}_R \cdot \bar{I} = (8)(10\angle -36.9^o) = 80\angle -36.9^o\ V$$

$$v_R(t) = 113.1 \cdot \cos(377t - 36.9^o)$$

$$\bar{V}_C = \bar{Z}_C \cdot \bar{I} = (-j4)(10\angle -36.9^o) = 40\angle -126.9^o\ V$$

$$v_C(t) = 56.57 \cdot \cos(377t - 126.9^o)$$

$$\bar{V}_L = \bar{Z}_L \cdot \bar{I} = (j10)(10\angle -36.9^o) = 100\angle 53.1^o\ V$$

$$v_L(t) = 141.4 \cdot \cos(377t + 53.1^o)$$

Absorbed powers $\bar{S} = \bar{V} \cdot \bar{I}^* = P + jQ$

$$\bar{S}_R = \bar{V}_R \cdot \bar{I}^* = 80\angle -36.9^o (10\angle -36.9^o)^* = 800 + j0$$

$$\bar{S}_C = \bar{V}_C \cdot \bar{I}^* = 40\angle -126.9^o (10\angle -36.9^o)^* = 0 - j400$$

$$\bar{S}_L = \bar{V}_L \cdot \bar{I}^* = 100\angle 53.1^o (10\angle -36.9^o)^* = 0 + j1000$$

$$\bar{S}_{TOT} = \bar{S}_R + \bar{S}_C + \bar{S}_L = 800 + j600$$

$$P_{TOT} = 800 \text{ watts}; \qquad Q_{TOT} = 600 \text{ vars};$$

$$S_{TOT} = |\bar{S}_{TOT}| = 1000\, VA$$

Delivered power

$$\bar{S}_S = \bar{V}_S \cdot \bar{I}^* = 100\angle 0^o (10\angle -36.9^o)^* = 800 + j600$$

$$\bar{S}_S = \bar{S}_{TOT} = 800 + j600$$

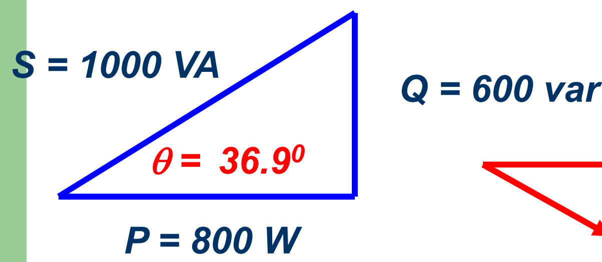
$$P_S = P_{TOT} = 800 \text{ watts}$$

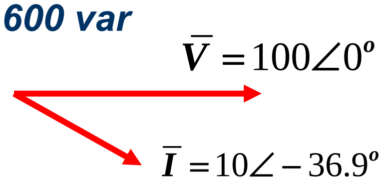
$$Q_S = Q_{TOT} = 600 \text{ vars}$$

In General: $P_{ABS} = P_{DEV}$ $\qquad Q_{ABS} = Q_{DEV}$

(Tellegen's Theorem)

The Power Triangle $\bar{S} = 800 + j600$

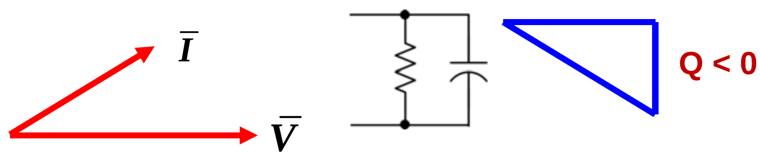




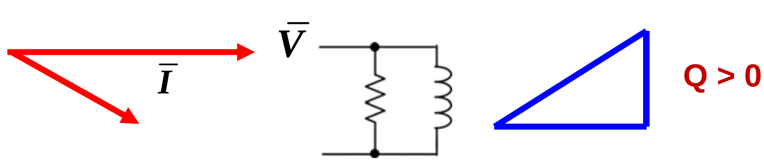
$$\text{power factor} = pf = \frac{P}{S} = \cos(\theta) = 0.8$$ ***lagging***

Leading, Lagging Concepts

Leading Case

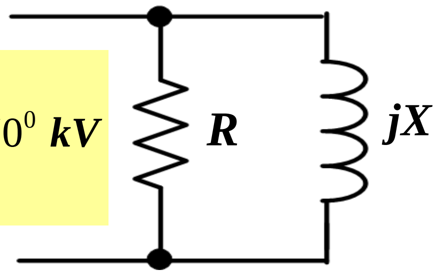


Lagging Case



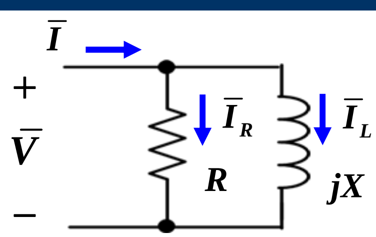
A Lagging pf Example

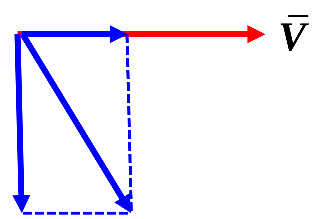
$$\bar{V} = 7.2\angle 0^0 \; kV$$



$R = 103.68\,\Omega$ $\qquad$ $jX = j43.2\,\Omega$

Currents





$$\bar{I}_R = \frac{\bar{V}}{R} = \frac{7.2}{103.68} = 69.44\; A$$

$$\bar{I}_L = \frac{\bar{V}}{jX} = \frac{7.2}{j43.20} = -j166.7\; A$$

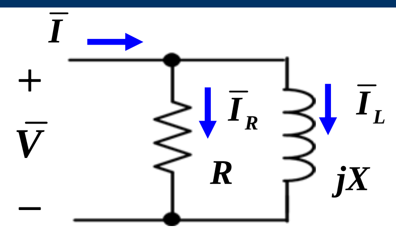
$$\bar{I} = \bar{I}_R + \bar{I}_L$$

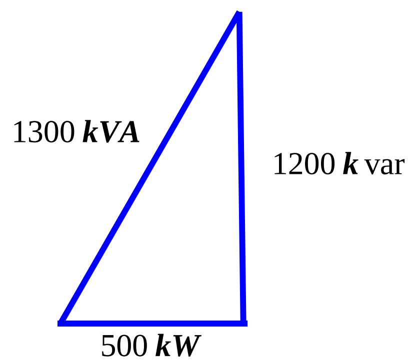
$$\bar{I} = 69.44 - j166.7$$

$$\bar{I} = 180.6\angle -67.38^0\; A$$

Powers

$pf = \cos(\theta) = \cos(67.36^0) = 0.3845$





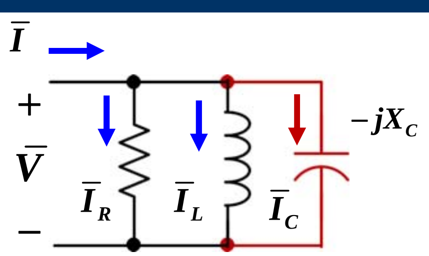
$\bar{S}_R = \bar{V} \cdot \bar{I}_R{}^* = 500\ kW + j0$

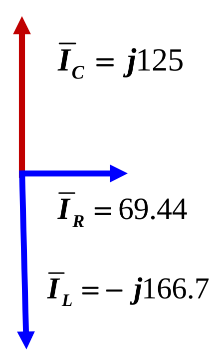
$\bar{S}_L = \bar{V} \cdot \bar{I}_L{}^* = 0 + j1200\ k\,\text{var}$

$\bar{S}_S = \bar{V} \cdot \bar{I}^* = \bar{S}_R + \bar{S}_L$

$\bar{S}_S = 500 + j1200 = 1300\angle 67.38^0$

Add Capacitance





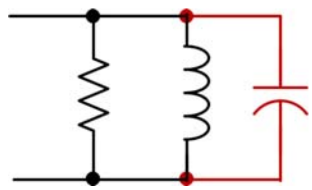
$\bar{I}_C = \dfrac{\bar{V}}{-jX} = \dfrac{7.2}{-j57.6} = j125\ A$

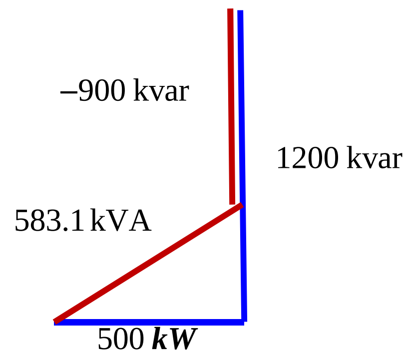
$\bar{I} = \bar{I}_R + \bar{I}_L + \bar{I}_C$

$\bar{I} = 69.44 - j166.7 + j125 = 81\angle -31^0\ A$

Powers

$$pf = \cos(\theta) = \cos(31^0) = 0.8575$$





$$\bar{S}_C = \bar{V} \cdot \bar{I}_C^* = 0 - j900 \text{ kvar}$$

$$\bar{S}_S = \bar{V} \cdot \bar{I}^* = \bar{S}_R + \bar{S}_L + \bar{S}_C$$

$$\bar{S}_S = 500 + j1200 - j900$$

$$\bar{S}_S = 583.1\angle 31^0 \; kVA$$

Observations

By adding capacitance to a lagging pf (inductive) load, we have significantly reduced the source current., *without changing P!*

Before $I = 180.6\ A; \quad pf = 0.3845$

After $I = 81\ A; \quad pf = 0.8575$

Note that: ***low pf, high current;***
high pf, low current;

If we consider the "source" in the example to represent an Electric Utility, this reduction in current is of major practical importance, since the utility losses are proportional to the *square* of the current.

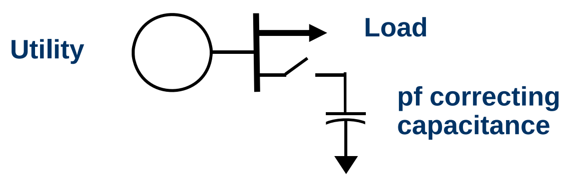
Observations

That is, by adding capacitance the utility losses have been reduced by almost ***a factor of 5!*** Since this results in significant savings to the utility, it has an incentive to induce its customers to operate at high pf.

This leads to the "**Power Factor Correction**" problem, which is a classic in electric power engineering and is extremely likely to be on the FE exam.

We will be using the same numerical data as we did in the previous example. Pretty clever, eh' what?

The Power Factor Correction problem



An Electric Utility supplies 7.2 kV to a customer whose load is 7.2 kV 1300 kVA @ pf = 0.3845 lagging. The utility offers the customer a reduced rate if he will "correct" ("improve" or "raise") his pf to 0.8575. Determine the requisite capacitance.

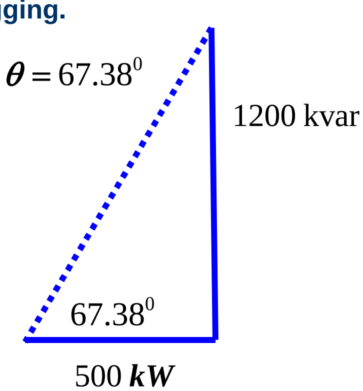
PF Correction: the solution

1. **Draw the load power triangle.**
   1300 kVA @ pf = 0.3845 lagging.

$$pf = 0.3845 = \cos(\theta) \qquad \theta = 67.38^0$$

$$\bar{S}_{LOAD} = S\angle\theta = 1300\angle 67.38^0$$

$$\bar{S}_{LOAD} = 500 + j1200$$



Because the pf is lagging, the load is inductive, and Q is positive. Therefore we must add negative Q to reduce the total, which means we must add capacitance.

PF Correction: the solution

2. We need to modify the source complex power so that the pf rises to 0.8575 lagging.

$$pf = 0.8575 = \cos(\theta) \qquad \theta = 31^0$$

Closing the switch (inserting the capacitors)

$$\bar{S}_S = 500 + j1200 - jQ_C = 500 + j(1200 - Q_C)$$

$$\text{Let } Q_X = 1200 - Q_C$$

$$\text{Therefore } \bar{S}_S = 500 + jQ_X = S_S\angle 31^0 \ kVA$$

$$\text{Then } \mathrm{Tan}(\theta) = \frac{Q_X}{500} = Tan(31^0) = 0.6$$

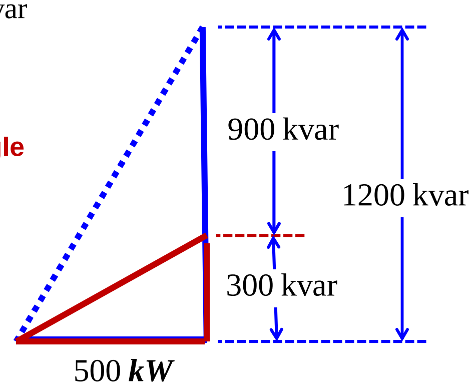
PF Correction: the solution

$$\frac{Q_X}{500} = 0.6 \qquad Q_X = 300 \text{ kvar}$$

$$Q_C = 1200 - Q_X = 900 \text{ kvar}$$

The new **source power triangle**

Install 900 kvar of 7.2 kV Capacitors



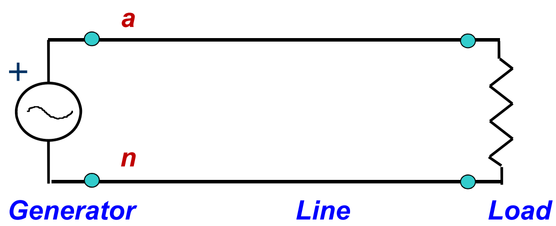
4. Three-phase ac Circuits

Although essentially all types of EE's use ac circuit analysis to some degree, the overwhelming majority of applications are in the high energy ("power") field.

It happens that if power levels are above about 10 kW, it is more practical and efficient to arrange ac circuits in a "polyphase" configuration. Although any number of "phases" are possible, "3-phase" is almost exclusively used in high power applications, since it is the simplest case that achieves most of the advantage of polyphase.

It is virtually certain that some 3-phase problems will appear on the FE and PE examinations, which is why 3-phase merits our attention.

A single-phase ac circuit

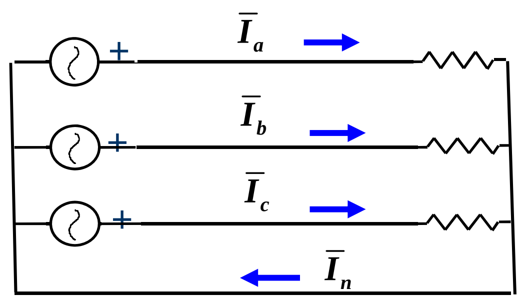


"a" is the "phase" conductor

"n" is the "neutral" conductor

For a given load, the phase a conductor must have a cross-sectional area "A", large enough to carry the requisite current. Since the neutral carries the return current, we need a total of "2A worth" of conductors.

Tripling the capacity



If $\bar{I}_a = \bar{I}_b = \bar{I}_c = I\angle\theta$ then $\bar{I}_n = 3I\angle\theta$

We need a total of A + A +A + 3A = 6A conductors.

But what if the currents are not in phase?

Suppose $\bar{I}_a = I\angle 0^0 \quad \bar{I}_b = I\angle -120^0 \quad \bar{I}_c = I\angle +120^0$

Then

$$\bar{I}_n = \bar{I}_a + \bar{I}_b + \bar{I}_c = I\angle 0^0 + I\angle -120^0 + I\angle +120^0$$

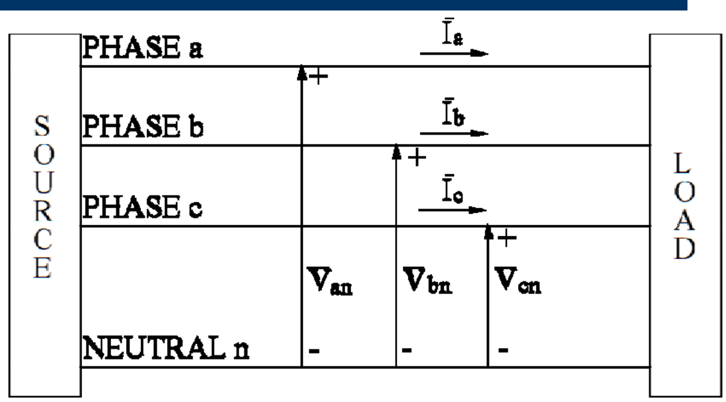
$$\bar{I}_n = I\left[(1+j0)+(-0.5-j0.866)+(-0.5+j0.866)\right]$$

$$\bar{I}_n = I\left[(1.0-0.5-0.5)+j(0.0-0.866+0.866)\right] = I(0+j0) = 0$$

Now we only need a total of A + A +A + 0 = 3A conductors!

A 50% savings!

The 3-Phase Situation

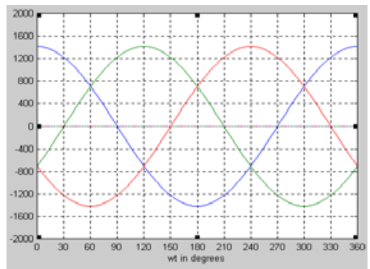


"PHASE" CONDUCTORS ARE ALSO CALLED "LINES"

"Balanced" voltage means equal in magnitude, 120º separated in phase

$$v_{an}(t) = V_{max}\cos(\omega t) = \sqrt{2}\cdot V\cdot\cos(\omega t)$$
$$v_{bn}(t) = V_{max}\cos(\omega t - 120^0) = \sqrt{2}\cdot V\cdot\cos(\omega t - 120^0)$$
$$v_{cn}(t) = V_{max}\cos(\omega t + 120^0) = \sqrt{2}\cdot V\cdot\cos(\omega t + 120^0)$$

$$\bar{V}_{an} = V\angle 0^0$$
$$\bar{V}_{bn} = V\angle -120^0$$
$$\bar{V}_{cn} = V\angle +120^0$$



The "Line" Voltages

By KVL $\bar{V}_{ab} = \bar{V}_{an} - \bar{V}_{bn}$

$$\bar{V}_{ab} = V\angle 0^0 - V\angle -120^0 = V\left[1 + j0 - \left(-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right)\right] = V\sqrt{3}\angle 30^0$$

$$\bar{V}_{bc} = V\sqrt{3}\angle -90^0$$
$$\bar{V}_{ca} = V\sqrt{3}\angle 150^0$$

$$V_{ab} = V_{bc} = V_{ca} = V_L = V\sqrt{3}$$

An Example

When a power engineer says "the primary distribution voltage is 12 kV" he/she means…

$$V_{ab} = V_{bc} = V_{ca} = V_L = 12.47\ kV$$

$$\bar{V}_{ab} = 12.47\angle 30^0\ kV$$
$$\bar{V}_{bc} = 12.47\angle -90^0\ kV$$
$$\bar{V}_{ca} = 12.47\angle +150^0\ kV$$

$$V_{an} = V_{bn} = V_{cn} = \frac{V_L}{\sqrt{3}} = 7.2\ kV$$

$$\bar{V}_{an} = 7.2\angle 0^0\ kV$$
$$\bar{V}_{bn} = 7.2\angle -120^0\ kV$$
$$\bar{V}_{cn} = 7.2\angle +120^0\ kV$$

An Important Insight….

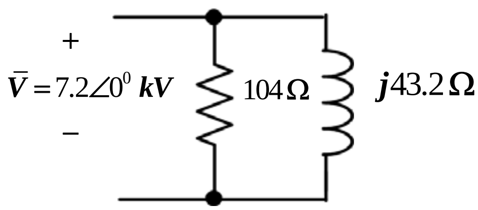
All balanced three-phase problems can be solved by focusing on a-phase, solving the single-phase (a-n) problem, and using 3-phase symmetry to deal with b-n and c-n values!

This involves judicious use of the factors 3, $\sqrt{3}$, and 120^0!

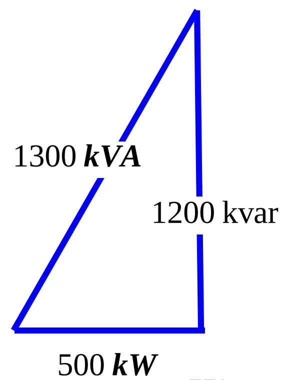
To demonstrate…

Recall the pf Correction Problem

An Electric Utility supplies 7.2 kV to a single-phase customer whose load is 7.2 kV 1300 kVA @ pf = 0.3845 lagging.

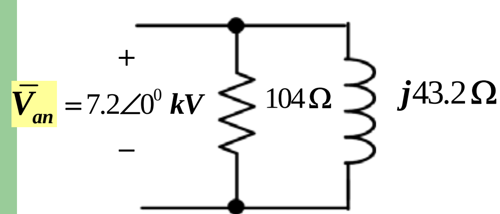


$$\bar{I} = \bar{I}_R + \bar{I}_L = 181\angle -67^0\ A$$

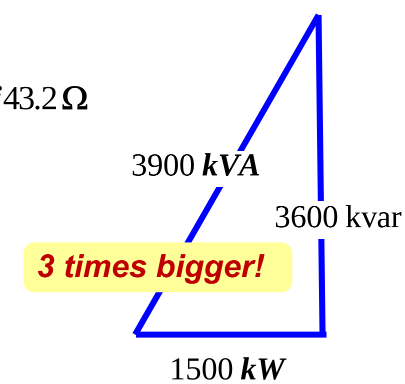


The pf Correction Problem in the 3-phase case

An Electric Utility supplies 12.47 kV to a 3-phase customer whose load is 12.47 kV 3900 kVA @ pf = 0.3845 lagging.



$$\bar{I}_a = 181\angle -67^0\ A$$



If we want all the V's, I's, and S's

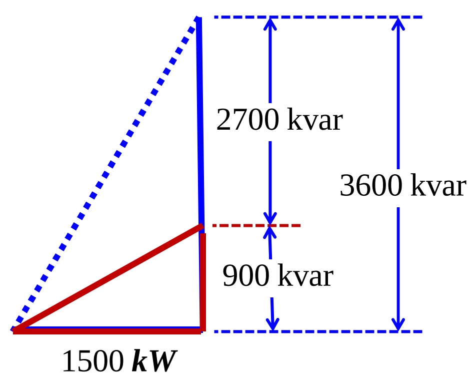
$$\bar{V}_{an} = 7.2\angle 0^0 \ \boldsymbol{kV} \qquad \bar{V}_{ab} = 12.47\angle 30^0 \ \boldsymbol{kV}$$
$$\bar{V}_{bn} = 7.2\angle -120^0 \ \boldsymbol{kV} \qquad \bar{V}_{bc} = 12.47\angle -90^0 \ \boldsymbol{kV}$$
$$\bar{V}_{cn} = 7.2\angle +120^0 \ \boldsymbol{kV} \qquad \bar{V}_{ca} = 12.47\angle +150^0 \ \boldsymbol{kV}$$

$$\bar{I}_a = 181\angle -67^0 \ \boldsymbol{A} \qquad \bar{S}_a = 500 + \boldsymbol{j}1200\, \boldsymbol{kVA}$$
$$\bar{I}_b = 181\angle -187^0 \ \boldsymbol{A} \qquad \bar{S}_b = 500 + \boldsymbol{j}1200\, \boldsymbol{kVA}$$
$$\bar{I}_c = 181\angle +53^0 \ \boldsymbol{A} \qquad \bar{S}_c = 500 + \boldsymbol{j}1200\, \boldsymbol{kVA}$$

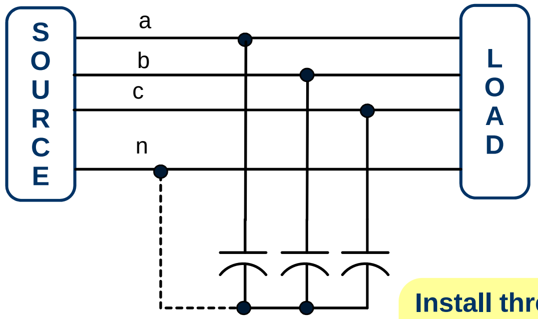
PF Correction: the 3ph solution

Install 2700 kvar of Capacitance.

The circuitry in the 3-phase case is a bit more complicated. There are two possibilities….

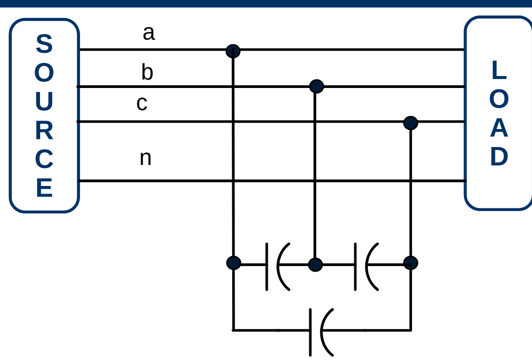


The wye connection….



Install three 900 kvar 7.2 kV wye-connected Capacitors.

The delta connection….



Install three 900 kvar 12.47 kV delta-connected Capacitors.

wye-delta connections

$$\bar{Z}_\Delta = 3 \cdot \bar{Z}_Y$$

wye case

$$Q_{an} = \frac{2700}{3} = 900 \text{ kvar}$$

$$I_a = \frac{Q_{an}}{V_{an}} = \frac{900}{7.2} = 125\ A$$

$$Z_{an} = Z_Y = \frac{V_{an}}{I_a} = 57.6\,\Omega$$

$$C_Y = \frac{1}{\omega Z_Y} = 46.05\ \mu F$$

delta case

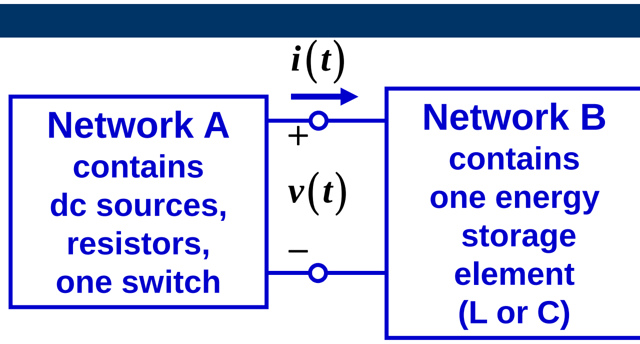
$$Q_{an} = \frac{2700}{3} = 900 \text{ kvar}$$

$$I_{ab} = \frac{Q_{ab}}{V_{ab}} = \frac{900}{12.47} = 72.17\ A$$

$$Z_{ab} = Z_\Delta = \frac{12.47}{72.17} = 172.8\,\Omega$$

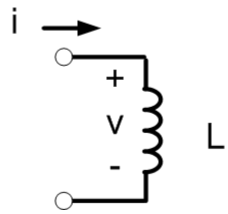
$$C_\Delta = \frac{1}{\omega Z_\Delta} = 15.35\ \mu F$$

1st Order Transients…..



The problem….(1) solve for v and/or i @ t < 0; (2) switch is switched @ t = 0; (3) solve for v and/or i for t > 0

The inductive case



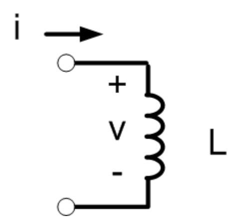
$$v_L = L \cdot \frac{di_L}{dt}$$

L's are SHORTS to dc

$i_L(t)$ cannot change in zero time

$$i_L(0^-) = i_L(0) = i_L(0^+)$$

An Example…



$$v_L = L \cdot \frac{di_L}{dt}$$

$$i_L(0^-) = i_L(0) = i_L(0^+)$$

Solution....

$t \le 0: \quad v_C(t) = v_C(0) \quad (\text{constant})$

$t \to \infty: \quad v_C(t) = v_C(\infty) \quad (\text{constant})$

$0 < t < \infty: \quad v_C(t) = v_C(\infty) + \left[v_C(0) - v_C(\infty)\right] \cdot e^{-t/\tau}$

$\tau = R_{ab} \cdot C$

Our job is to determine

$$v_C(0); \quad v_C(\infty); \quad \text{and}\, \tau = R_{ab} \cdot C$$

Solution....

For a capacitor: $$i_C = C \cdot \frac{dv_C}{dt}$$

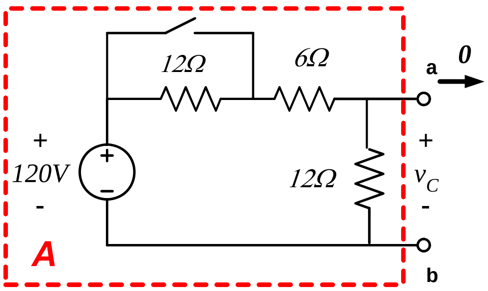
C's are OPENS to dc

$v_C(t)$ cannot change in zero time

Therefore, if the circuit is switched at t = 0:

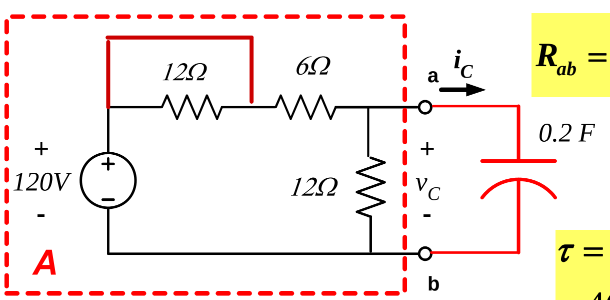
$$v_C(0^-) = v_C(0) = v_C(0^+)$$

Solution: T < 0; switch and "C" OPEN



$$v_C(0) = \frac{120}{12+6+12}(12) = 48\,V$$

Solution: T > 0; switch CLOSED



$$R_{ab} = \frac{6 \cdot 12}{6+12} = 4\,\Omega$$

$$\tau = R_{ab}C = 4(0.2) = 0.8\,s$$

$$v_C(\infty) = \frac{120}{0+6+12}(12) = 80\,V$$

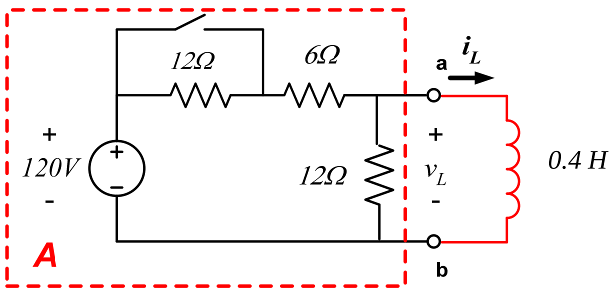
Solution....

$$v_C(0) = 48 \quad v_C(\infty) = 80$$

$$t > 0: \qquad v_C(t) = 80 + (48 - 80) \cdot e^{-1.25t}$$

$$v_C(t) = 80 - 32 \cdot e^{-1.25t}$$

3. 1st Order Transients: RL



b. The switch is closed at t = 0. Find and plot $i_L(t)$.

Solution....

$t \le 0: \quad i_L(t) = i_L(0)$

$t > 0: \quad i_L(t) = i_L(\infty) + \left[i_L(0) - i_L(\infty) \right] \cdot e^{-t/\tau}$

$$\tau = \frac{L}{R_{ab}}$$

Our job is to determine

$$i_L(0); \quad i_L(\infty); \quad \text{and}\, \tau = L / R_{ab}$$

Solution....

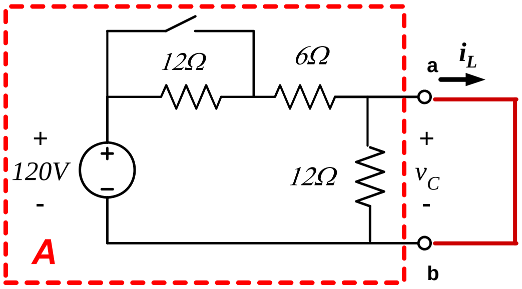
For an inductor: $v_L = L \cdot \dfrac{di_L}{dt}$

L's are SHORTS to dc

$i_L(t)$ cannot change in zero time

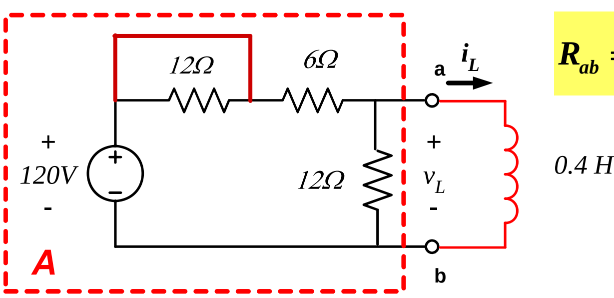
$$i_L(0^-) = i_L(0) = i_L(0^+)$$

Solution: T < 0; switch OPEN; L SHORT



$$i_L(0) = \frac{120}{12+6} = 6.667\ A$$

Solution: T > 0; switch CLOSED



$$R_{ab} = \frac{6 \cdot 12}{6+12} = 4\ \Omega$$

$$\tau = \frac{L}{R_{ab}} = \frac{0.4}{4} = 0.1\ s$$

$$i_L(\infty) = \frac{120}{0+6+0} = 20\ A$$

Solution....

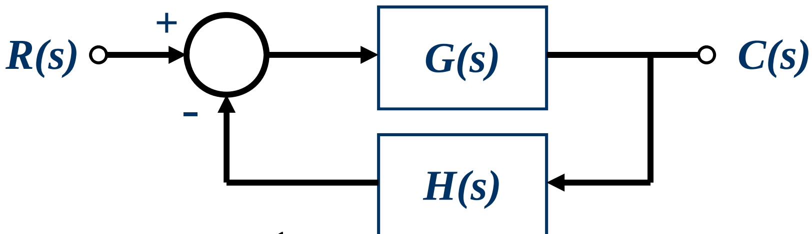
$t \le 0: \quad i_L(t) = 6.667$

$t > 0: \quad i_L(t) = 20 + (6.667 - 20) \cdot e^{-t/\tau}$

$$i_L(t) = 20 - 13.33 \cdot e^{-10t}$$

5. Control

Given the following feedback control system:



$$G(s) = \frac{1}{(s-1)(s+4)} \qquad H(s) = K$$

a. Write the closed loop transfer function in rational form

$$\frac{C}{R} = \frac{G}{1+GH} = \frac{\dfrac{1}{(s-1)(s+4)}}{1+\dfrac{K}{(s-1)(s+4)}}$$

$$\frac{C}{R} = \frac{1}{(s-1)(s+4)+K} = \frac{1}{s^2+3s+(K-4)}$$

b. Write the characteristic equation

$$s^2+3s+(K-4)=0$$

c. What is the system order? **2**

d. For K = 0, where are the poles located?

$$s^2+3s-4=(s-1)\cdot(s+4)=0$$

s = +1; s = - 4

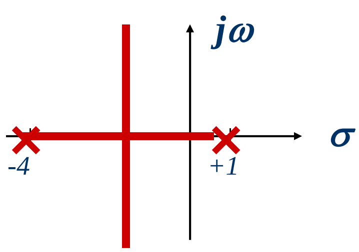
e. For K = 0, is the system stable? **NO**

f. Complete the table $s^2 + 3s + (K - 4) = 0$

Roots of the CE are poles of the CLTF

K	poles	damping
0	$-4.00, +1.00$	unstable
4	$-3.00, 0.00$	over
5	$-2.62, -0.382$	over
6.25	$-1.50, -1.50$	critical
10.25	$-1.5 - j2, -1.5 + j2$	under

f. Sketch the root locus



g. Find the range on K for system stability.

If K = 4:

$$s^2 + 3s + 0 = (s)\cdot(s+3) = 0$$

Poles at s = 0; s = - 3

Therefore for K>4, poles are in LH s-plane and system is stable.

$K \geq 4$

h. Find K for critical damping

$$CE: \quad s^2 + 3s + (K-4) = 0$$

Solving the CE: $$s = \frac{-3 \pm \sqrt{9-4(K-4)}}{2}$$

Critical damping occurs when the poles are real and equal

$$\sqrt{9-4(K-4)} = 0$$

$$K - 4 = 9/4;$$

$$K = 4 + 2.25 = 6.25$$

6. Signal Processing

a. periodic time-domain functions have
continuous (discrete) frequency spectra.
(circle the correct adjective)

b. aperiodic time-domain functions have
(continuous) discrete frequency spectra.
(circle the correct adjective)

c. Matching

Laplace Transform	d	*Fourier Transform*	c
Fourier Series	a	*Convolution integral*	b
Inverse FT	e		

a. $x(t) = \sum_{n=-N}^{N} \bar{D}_n \exp(jn\omega_0 t)$

b. $y(t) = \int_{-\infty}^{t} x(\tau) \cdot h(t-\tau) \cdot d\tau$

c. $\bar{X}(j\omega) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j\omega t} \cdot dt$

d. $X(s) = \int_{0}^{\infty} x(t) \cdot e^{-st} \cdot dt$

e. $x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \bar{X}(j\omega) \cdot e^{+j\omega t} \cdot d\omega$

c. Matching

Z-Transform	d	*DFT*	c
Inverse ZT	a	*Discrete Convolution*	b
Inverse DFT	e		

a. $\bar{X}(j\Omega) = \sum_{n=-\infty}^{\infty} x[n] \cdot e^{-jn\Omega}$

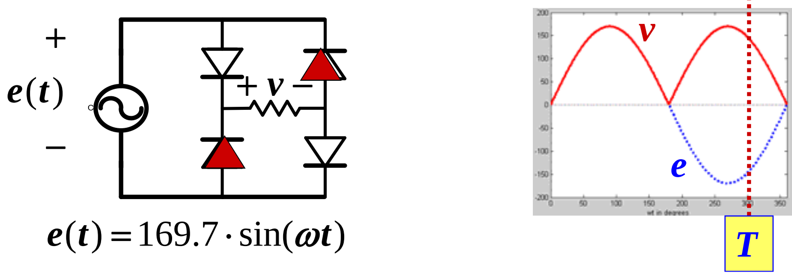
b. $y[k] = \sum_{n=-\infty}^{k} x[n] \cdot h[n-k]$

c. $\bar{X}_k = \sum_{n=0}^{N-1} x[n] \cdot e^{-j2\pi kn/N}$

d. $X(z) = \sum_{n=0}^{\infty} x[n] \cdot z^{-n}$

e. $x[n] = \frac{1}{N} \sum_{k=0}^{N-1} \bar{X}_k \cdot e^{+j2\pi kn/N}$

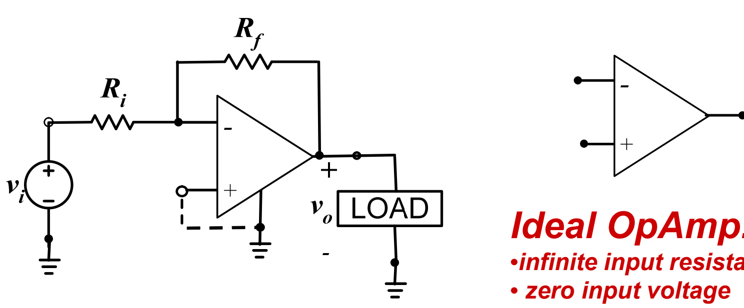
7. Electronics



$$e(t) = 169.7 \cdot \sin(\omega t)$$

a. Darken the conducting diodes at time T

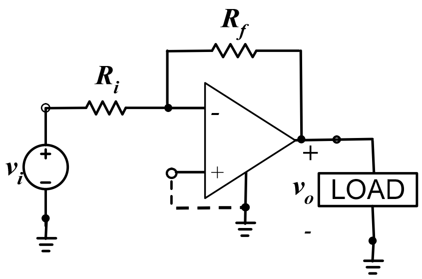
b. Given the "OP Amp" circuit



Ideal OpAmp....

- infinite input resistance
- zero input voltage
- infinite gain
- zero output resistance

Find the output voltage.



$v_i = 5\,V$

$R_i = 10\,k\Omega$

$R_f = 50\,k\Omega$

$$KCL: \frac{v_i}{R_i} + \frac{v_0}{R_f} = 0$$

$$v_0 = -\left(\frac{R_f}{R_i}\right) \cdot v_i$$

$$v_0 = -\left(\frac{50}{10}\right) \cdot 5 = -25\,V$$

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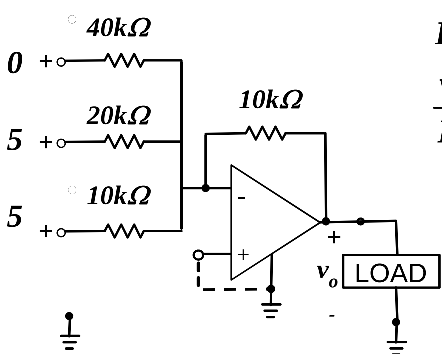
c. Find the output voltage.



$KCL:$

$$\frac{v_1}{R_1} + \frac{v_2}{R_2} + \frac{v_3}{R_3} + \frac{v_0}{R_f} = 0$$

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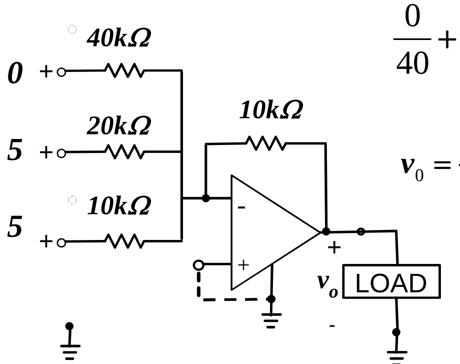
Solution: "SUMMER"



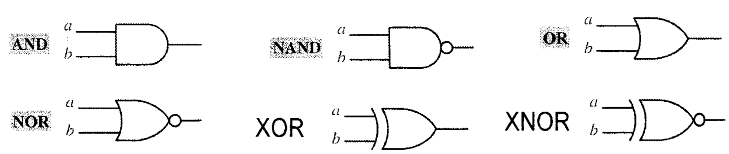
$$\frac{0}{40}+\frac{5}{20}+\frac{5}{10}+\frac{v_0}{10}=0$$

$$v_0=-\left(\frac{10}{10}\right)\cdot 5-\left(\frac{10}{20}\right)\cdot 5-\left(\frac{10}{40}\right)\cdot 0$$

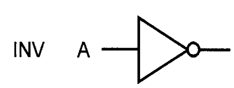
$$v_0=-7.5\,V$$

8. Digital Systems Logic Gates

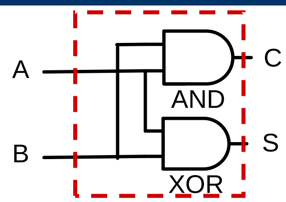
A	B	AND	NAND	OR	NOR	XOR	XNOR
0	0	0	1	0	1	0	1
0	1	0	1	1	0	1	0
1	0	0	1	1	0	1	0
1	1	1	0	1	0	0	1



A	INV
0	1
1	0



a. Complete the Truth Table



Half Adder (HA)

A	B	C	S
0	0	**0**	**0**
0	1	**0**	**1**
1	0	**0**	**1**
1	1	**1**	**0**

A + B = CS

0 + 0 = 00

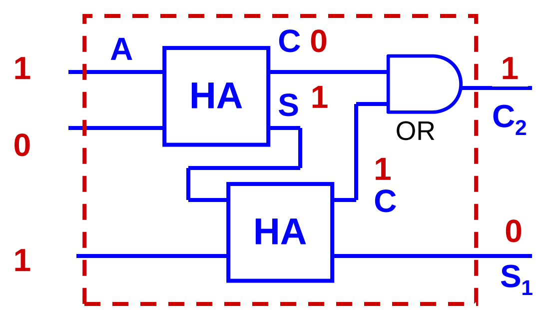
0 + 1 = 01

1 + 0 = 01

1 + 1 = 10

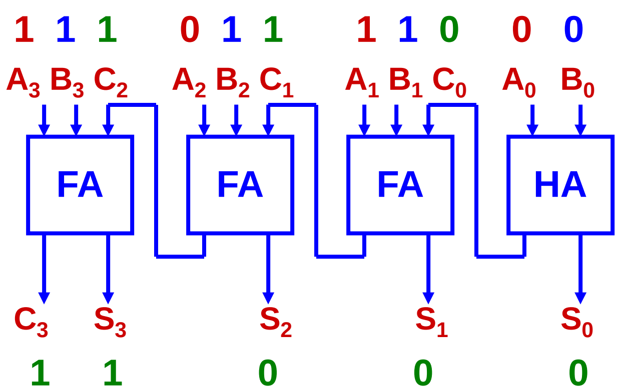
b. Complete the indicated row in the TT

C_1	A_1	B_1	C_2	S_1
0	0	0	**0**	**0**
0	0	1	**0**	**1**
0	1	0	**0**	**1**
0	1	1	**1**	**0**
1	0	0	**0**	**1**
1	0	1	**1**	**0**
1	1	0	**1**	**0**
1	1	1	**1**	**1**



Full Adder (FA)

c. Indicate the inputs and outputs to perform the given sum in a 4-bit adder



1010
+1110

11000

d. Design a D/A Converter to accomodate 3-bit digital inputs (5 volt logic)

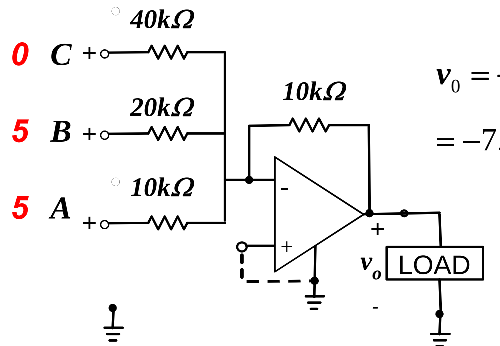
Resolution: 3-bits
(2^3 = 8 levels;
10 V scale)

Example...
Convert "110"
to analog

Digital	*Analog (V)*
000	*0.00*
001	*1.25*
010	*2.50*
011	*3.75*
100	*5.00*
101	*6.25*
110	*7.50*
111	*8.75*

Binary Word: ABC
(A msb; C lsb)

d. Finished Design ABC = 110



$$v_0 = -\frac{10}{10}\cdot 5 - \frac{10}{20}\cdot 5 - \frac{10}{40}\cdot 0$$
$$= -7.50$$

Good Luck on the Exam!

If I can help with any ECE material, come see me (7:30 - 11:00; 1:15 - 2:30)

Charles A. Gross, Professor Emeritus
Electrical and Comp Engineering
Auburn University Broun 212
334.844.1812
gross@eng.auburn.edu

Good Evening...